

# Prelude

*The constant demands of my department and university and the ever increasing work needed to obtain funding have stolen much of my precious thinking time, and I sometimes yearn for the halcyon days of Bell Labs.*

–Steven Chu, Nobel laureate [79]

Convex Analysis is the calculus of inequalities while Convex Optimization is its application. Analysis is inherently the domain of a mathematician while Optimization belongs to the engineer. Practitioners in the art of Convex Optimization engage themselves with discovery of which hard problems, perhaps previously believed nonconvex, can be transformed into convex equivalents; because once convex form of a problem is found, then a globally optimal solution is close at hand – the hard work is finished: Finding convex expression of a problem is itself, in a very real sense, the solution.

There is a great race under way to determine which important problems can be posed in a convex setting. Yet, that skill acquired by understanding the geometry and application of Convex Optimization will remain more an art for some time to come; the reason being, there is generally no unique transformation of a given problem to its convex equivalent. This means, two researchers pondering the same problem are likely to formulate a convex equivalent differently; hence, one solution is likely different from the other for the same problem, and any convex combination of those two solutions remains optimal. Any presumption of only one right or correct solution becomes nebulous. Study of equivalence & sameness, uniqueness, and duality therefore pervade study of Optimization.

Tremendous benefit accrues when an optimization problem can be transformed to its convex equivalent, primarily because any locally optimal solution is then guaranteed globally optimal. Solving a nonlinear system, for example, by instead solving an equivalent convex optimization problem is therefore highly preferable.<sup>0.1</sup> Yet it can be difficult for the engineer to apply theory without an understanding of Analysis.

These pages comprise my journal over an eight year period bridging gaps between engineer and mathematician; they constitute a translation, unification, and cohering of about three hundred papers, books, and reports from several different fields of mathematics and engineering. Beacons of historical accomplishment are cited throughout. Much of what is written here will not be found elsewhere. Care to detail, clarity, accuracy, consistency, and typography accompanies removal of ambiguity and verbosity out of respect for the reader. Consequently there is much cross-referencing and background material provided in the text, footnotes, and appendices so as to be self-contained and to provide understanding of fundamental concepts.

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<sup>0.1</sup>That is what motivates a convex optimization known as *geometric programming*, invented in 1960s, [58] [77] which has driven great advances in the electronic circuit design industry. [33, §4.7] [243] [376] [379] [98] [176] [186] [187] [188] [189] [190] [258] [259] [303]