Appendix F

Notation, Definitions, Glossary

b	(italic $abcdefghijklmnopqrstuvwxyz$) column vector, scalar, or logical condition
b_i	i^{th} *entry of vector* $b=[b_i\,,~i=1\ldots n]$ or i^{th} b *vector from a set or list* $\{b_j\,,~j=1\ldots n\}$ or i^{th} *iterate of vector* b
$b_{i:j}$	or $b(i:j)$, truncated vector comprising i^{th} through j^{th} entry of vector b
$b_k(i:j)$	or $b_{i:j,\,k}$, truncated vector comprising i^{th} through j^{th} entry of vector b_k
$b^{\mathrm T}$	vector transpose or row vector
$b^{\mathrm H}$	complex conjugate transpose $b^{*\mathrm T}$
A	matrix, scalar, or logical condition (italic $ABCDEFGHIJKLMNOPQRSTUVWXYZ$)
$A^{\mathrm T}$	Matrix transpose $[A_{ij}]\leftarrow[A_{ji}]$ is a linear operator. Regarding A as a linear operator, $A^{\mathrm T}$ is its adjoint.
$A^{-\mathrm T}$	matrix transpose of inverse; and *vice versa,* $\left(A^{-1}\right)^{\mathrm T}=\left(A^{\mathrm T}\right)^{-1}$
$A^{\mathrm T_1}$	first of various transpositions of a cubix or quartix A (p.584, p.588)
skinny	a skinny matrix; meaning, more rows than columns: $\left[\begin{array}{c} \\ \\ \\ \end{array}\right]$. *When there are more equations than unknowns, we say that the system* $Ax=b$ *is overdetermined.* [174, §5.3]
fat	a fat matrix; meaning, more columns than rows: $[\qquad\qquad]$. *underdetermined*
$\mathcal A$	some set (calligraphic $\mathcal{ABCDEFGHIJKLMNOPQRSTUVWXYZ}$)

$\mathcal{F}(\mathcal{C} \ni A)$	smallest face (171) that contains element A of set $\mathcal{C}$
$\mathfrak{F}$	discrete Fourier transform (891)
$\mathcal{G}(\mathcal{K})$	generators (§2.13.4.2.1) of set $\mathcal{K}$; any collection of points and directions whose hull constructs $\mathcal{K}$
$\mathcal{L}_\nu^\nu$	level set (556)
$\mathcal{L}_\nu$	sublevel set (560)
$\mathcal{L}^\nu$	superlevel set (646)
A^{-1}	inverse of matrix A
$A^\dagger$	Moore-Penrose pseudoinverse of matrix A
$\sqrt{\ }$	positive square root
$\sqrt[\circ]{x}$	entrywise positive square root of vector x
$\sqrt[\ell]{\ }$	positive ℓ^{th} root
$A^{1/2}$ and $\sqrt{A}$	$A^{1/2}$ is any matrix such that $A^{1/2}A^{1/2} = A$. For $A \in \mathbb{S}_+^n$, $\sqrt{A} \in \mathbb{S}_+^n$ is unique and $\sqrt{A}\sqrt{A} = A$. [56, §1.2] (§A.5.1.3)
$\sqrt[\circ]{D}$	$= [\sqrt{d_{ij}}\,]$. (1436) *Hadamard positive square root:* $D = \sqrt[\circ]{D} \circ \sqrt[\circ]{D}$
$\mathfrak{A}$	Euler Fraktur $\mathfrak{A}\mathfrak{B}\mathfrak{C}\mathfrak{D}\mathfrak{E}\mathfrak{F}\mathfrak{G}\mathfrak{H}\mathfrak{I}\mathfrak{J}\mathfrak{K}\mathfrak{L}\mathfrak{M}\mathfrak{N}\mathfrak{O}\mathfrak{P}\mathfrak{Q}\mathfrak{R}\mathfrak{S}\mathfrak{T}\mathfrak{U}\mathfrak{V}\mathfrak{W}\mathfrak{X}\mathfrak{Y}\mathfrak{Z}$
$\mathfrak{L}$	Lagrangian (506)
$\mathfrak{E}$	member of elliptope $\mathcal{E}_t$ (1185) parametrized by scalar t
$\mathcal{E}$	elliptope (1164)
E	elementary matrix
E_{ij}	member of standard orthonormal basis for symmetric (59) or symmetric hollow (75) matrices
A_{ij}	or $A(i\,,j)$, ij^{th} entry of matrix $A = \begin{array}{cc} \begin{array}{ccc} {\scriptstyle 1} & {\scriptstyle 2} & {\scriptstyle 3} \end{array} & \\ \left[\begin{array}{ccc} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{array}\right] & \begin{array}{c} {\scriptstyle 1} \\ {\scriptstyle 2} \\ {\scriptstyle 3} \end{array} \end{array}$ or rank-one matrix $a_i a_j^{\mathrm{T}}$ (§4.8)
$A(i\,,j)$	A is a function of i and j
A_i	i^{th} matrix from a set or i^{th} principal submatrix or i^{th} iterate of A
$A(i\,,:)$	i^{th} row of matrix A
$A(:\,,j)$	j^{th} column of matrix A [174, §1.1.8]

$A_{i:j,\,k:\ell}$	or $A(i\!:\!j\,,k\!:\!\ell)$, submatrix taken from i^{th} through j^{th} row and k^{th} through ℓ^{th} column
id est	from the Latin meaning *that is*
e.g	*exempli gratia,* from the Latin meaning *for sake of example*
videlicet	from the Latin meaning *it is permitted to see*
no.	*number,* from the Latin *numero*
a.i.	affinely independent (§2.4.2.3)
c.i.	conically independent (§2.10)
l.i.	linearly independent
w.r.t	*with respect to*
`a.k.a`	*also known as*
re	real part
im	imaginary part
$\imath$ or $\jmath$	$\sqrt{-1}$
$\subset \quad \supset \quad \cap \quad \cup$	standard set theory, *subset, superset, intersection, union*
$\in$	membership, *element belongs to,* or *element is a member of*
$\ni$	membership, *contains* as in $\mathcal{C} \ni y$ ($\mathcal{C}$ contains element y)
$\backepsilon$	*such that*
$\exists$	*there exists*
$\therefore$	*therefore*
$\forall$	*for all,* or *over all*
&	(ampersand) *and*
&	(ampersand italic) *and*
$\propto$	*proportional to*
∞	infinity
$\equiv$	*equivalent to*
$\triangleq$	*defined equal to, equal by definition*
$\approx$	*approximately equal to*

$\simeq$	*isomorphic to* or *with*
$\cong$	*congruent to* or *with*
—	Hadamard quotient as in, for $x\,,y\in\mathbb{R}^n$, $\dfrac{x}{y} \triangleq [\,x_i/y_i\ ,\ i=1\ldots n\,]\in\mathbb{R}^n$
$\circ$	Hadamard product of matrices: $x\circ y \triangleq [\,x_i\,y_i\ ,\ i=1\ldots n\,]\in\mathbb{R}^n$
$\otimes$	Kronecker product of matrices (§D.1.2.1)
$\oplus$	vector sum of sets $\mathcal{X}=\mathcal{Y}\oplus\mathcal{Z}$ where every element $x\in\mathcal{X}$ has unique expression $x=y+z$ where $y\in\mathcal{Y}$ and $z\in\mathcal{Z}$; [325, p.19] then summands are *algebraic complements.* $\mathcal{X}=\mathcal{Y}\oplus\mathcal{Z}\Rightarrow\mathcal{X}=\mathcal{Y}+\mathcal{Z}$. Now assume $\mathcal{Y}$ and $\mathcal{Z}$ are nontrivial subspaces. $\mathcal{X}=\mathcal{Y}+\mathcal{Z}\ \Rightarrow\ \mathcal{X}=\mathcal{Y}\oplus\mathcal{Z}\ \Leftrightarrow\ \mathcal{Y}\cap\mathcal{Z}=\mathbf{0}$ [326, §1.2] [116, §5.8]. Each element from a vector sum (+) of subspaces has unique expression ($\oplus$) when a basis from each subspace is linearly independent of bases from all the other subspaces.
$\ominus$	likewise, the vector difference of sets
$\boxplus$	orthogonal vector sum of sets $\mathcal{X}=\mathcal{Y}\boxplus\mathcal{Z}$ where every element $x\in\mathcal{X}$ has unique orthogonal expression $x=y+z$ where $y\in\mathcal{Y}$, $z\in\mathcal{Z}$, and $y\perp z$. [347, p.51] $\mathcal{X}=\mathcal{Y}\boxplus\mathcal{Z}\Rightarrow\mathcal{X}=\mathcal{Y}+\mathcal{Z}$. If $\mathcal{Z}\subseteq\mathcal{Y}^\perp$ then $\mathcal{X}=\mathcal{Y}\boxplus\mathcal{Z}\ \Leftrightarrow\ \mathcal{X}=\mathcal{Y}\oplus\mathcal{Z}$. [116, §5.8] If $\mathcal{Z}=\mathcal{Y}^\perp$ then summands are *orthogonal complements.*
$\pm$	*plus or minus* or *plus and minus*
$\perp$	as in $A\perp B$ meaning A *is orthogonal to* B (and *vice versa*), where A and B are sets, vectors, or matrices. When A and B are vectors (or matrices under Frobenius' norm), $A\perp B\ \Leftrightarrow\ \langle A\,,B\rangle=0\Leftrightarrow\ \|A+B\|^2=\|A\|^2+\|B\|^2$
$\backslash$	as in $\backslash\mathcal{A}$ means logical *not* $\mathcal{A}$, or *relative complement of set* $\mathcal{A}$; *id est*, $\backslash\mathcal{A}=\{x\notin\mathcal{A}\}$; *e.g*, $\mathcal{B}\backslash\mathcal{A}\triangleq\{x\in\mathcal{B}\mid x\notin\mathcal{A}\}\equiv\mathcal{B}\cap\backslash\mathcal{A}$
$\Rightarrow$ or $\Leftarrow$	sufficient or necessary, *implies,* or *is implied by*; *e.g*, A is sufficient: $A\Rightarrow B$, A is necessary: $A\Leftarrow B$, $A\Rightarrow B\ \Leftrightarrow\ \backslash A\Leftarrow\backslash B$, $A\Leftarrow B\ \Leftrightarrow\ \backslash A\Rightarrow\backslash B$, *if* A *then* B, *if* B *then* A, A *only if* B. B *only if* A.
$\Leftrightarrow$	*if and only if* (iff) or *corresponds with* or *necessary and sufficient* or *logical equivalence*
is	as in A *is* B means $A\Rightarrow B$; conventional usage of English language imposed by logicians
$\nRightarrow$ or $\nLeftarrow$	insufficient or unnecessary, *does not imply,* or *is not implied by*; *e.g*, $A\nRightarrow B\ \Leftrightarrow\ \backslash A\nLeftarrow\backslash B$. $A\nLeftarrow B\ \Leftrightarrow\ \backslash A\nRightarrow\backslash B$.
$\leftarrow$	*is replaced with*; substitution, assignment
$\rightarrow$	*goes to,* or *approaches,* or *maps to*

$t \to 0^+$	*t goes to* 0 *from above*; meaning, *from the positive* [215, p.2]
$\vdots \quad \ddots \quad \cdots$	as in $1\cdots1$ and $[\,s_1\cdots s_N\,]$ meaning *continuation*; respectively, ones in a row and a matrix whose columns are s_i for $i=1\ldots N$
$\ldots$	as in $i=1\ldots N$ meaning, i is a *sequence* of successive integers beginning with 1 and ending with N; *id est*, $1\ldots N = 1\!:\!N$
$:$	as in $f:\mathbb{R}^n\to\mathbb{R}^m$ meaning *f is a mapping,* or sequence of successive integers specified by bounds as in $i\!:\!j = i\ldots j$ (if $j<i$ then sequence is descending)
$f:\mathcal{M}\to\mathcal{R}$	meaning *f is a mapping from ambient space $\mathcal{M}$ to ambient $\mathcal{R}$*, not necessarily denoting either domain or range
$\mid$	as in $f(x)\mid x\in\mathcal{C}$ means *with the condition(s)* or *such that* or *evaluated for,* or as in $\{f(x)\mid x\in\mathcal{C}\}$ means *evaluated for each and every x belonging to set $\mathcal{C}$*
$g\vert_{x_{\rm p}}$	*expression g evaluated at* $x_{\rm p}$
$A\,,B$	as in, for example, $A\,,B\in\mathbb{S}^N$ means $A\in\mathbb{S}^N$ and $B\in\mathbb{S}^N$
$(A\,,B)$	*open interval* between A and B in $\mathbb{R}$, or *variable pair* perhaps of disparate dimension
$[A\,,B\,]$	*closed interval* or *line segment* between A and B in $\mathbb{R}$
$(\;)$	*hierarchal, parenthetical, optional*
$\{\;\}$	curly braces denote a *set* or *list, e.g,* $\{Xa \mid a\succeq0\}$ *the set comprising Xa evaluated for each and every* $a\succeq0$ where membership of a to some space is implicit, a *union*
$\langle\;\rangle$	angle brackets denote *vector inner-product* (33) (38)
$[\;]$	matrix or vector, or quote insertion, or citation
$[d_{ij}]$	matrix whose $ij^{\rm th}$ entry is d_{ij}
$[x_i]$	vector whose $i^{\rm th}$ entry is x_i
$x_{\rm p}$	particular value of x
x_0	particular instance of x, or initial value of a sequence x_i
x_1	first entry of vector x, or first element of a set or list $\{x_i\}$
x_ε	*extreme point*
x_+	vector x whose negative entries are replaced with 0; $x_+ = \frac{1}{2}(x+\vert x\vert)$ (534) or *clipped vector x* or *nonnegative part of x*
x_-	$x_- \triangleq \frac{1}{2}(x-\vert x\vert)$ or *nonpositive part of* $x = x_+ + x_-$

$\check{x}$	known data
$x^\star$	optimal value of variable x . optimal ⇒ feasible
x^*	*complex conjugate* or *dual variable* or *extreme direction of dual cone*
f^*	*convex conjugate function* $f^*(s) = \sup\{\langle s\,, x\rangle - f(x) \mid x \in \operatorname{dom} f\}$
$P_{\mathcal{C}}x$ or Px	projection of point x on set $\mathcal{C}$, P is operator or idempotent matrix
$P_k x$	projection of point x on set $\mathcal{C}_k$ or on range of implicit vector
$\delta(A)$	(`a.k.a` diag(A) , §A.1) *vector made from main diagonal of* A if A is a matrix; otherwise, *diagonal matrix made from vector* A
$\delta^2(A)$	$\equiv \delta(\delta(A))$. For vector or diagonal matrix Λ , $\delta^2(\Lambda) = \Lambda$
$\delta(A)^2$	$= \delta(A)\delta(A)$ where A is a vector
$\lambda_i(X)$	i^{th} entry of vector λ is function of X
$\lambda(X)_i$	i^{th} entry of vector-valued function of X
$\lambda(A)$	*vector of eigenvalues of matrix* A , (1550) typically arranged in nonincreasing order
$\sigma(A)$	*vector of singular values of matrix* A (always arranged in nonincreasing order), or *support function in direction* A
Σ	diagonal matrix of singular values, not necessarily square
$\sum$	sum
$\pi(\gamma)$	nonlinear *permutation operator* (or *presorting function*) arranges vector γ into nonincreasing order (§7.1.3)
Ξ	permutation matrix
Π	doublet or permutation operator or matrix
$\prod$	product
$\psi(Z)$	signum-like *step function* that returns a scalar for matrix argument (746), it returns a vector for vector argument (1668)
D	symmetric hollow matrix of distance-square or *Euclidean distance matrix*
$\mathbf{D}$	Euclidean distance matrix operator
$\mathbf{D}^{\mathrm{T}}(X)$	adjoint operator
$\mathbf{D}(X)^{\mathrm{T}}$	transpose of $\mathbf{D}(X)$
$\mathbf{D}^{-1}(X)$	inverse operator
$\mathbf{D}(X)^{-1}$	inverse of $\mathbf{D}(X)$

$D^\star$	optimal value of variable D
D^*	dual to variable D
D°	polar variable D
∂	*partial derivative* or *partial differential* or *matrix of distance-square squared* (1476) or *boundary* of set $\mathcal{K}$ as in $\partial\mathcal{K}$ (17) (24)
$\sqrt{d_{ij}}$	(absolute) distance scalar
d_{ij}	distance-square scalar, EDM entry
$\mathbf{V}$	geometric centering operator, $\mathbf{V}(D) = -VDV\frac{1}{2}$ (1084)
$\mathbf{V}_{\mathcal{N}}$	$\mathbf{V}_{\mathcal{N}}(D) = -V_{\mathcal{N}}^{\mathrm{T}} D V_{\mathcal{N}}$ (1098)
V	$N \times N$ symmetric elementary, auxiliary, projector, geometric centering matrix, $\mathcal{R}(V) = \mathcal{N}(\mathbf{1}^{\mathrm{T}})$, $\mathcal{N}(V) = \mathcal{R}(\mathbf{1})$, $V^2 = V$ (§B.4.1)
$V_{\mathcal{N}}$	$N \times N-1$ Schoenberg auxiliary matrix $\mathcal{R}(V_{\mathcal{N}}) = \mathcal{N}(\mathbf{1}^{\mathrm{T}})$, $\mathcal{N}(V_{\mathcal{N}}^{\mathrm{T}}) = \mathcal{R}(\mathbf{1})$ (§B.4.2)
$V_{\mathcal{X}}$	$V_{\mathcal{X}} V_{\mathcal{X}}^{\mathrm{T}} \equiv V^{\mathrm{T}} X^{\mathrm{T}} X V$ (1276)
X	point list ((76) having cardinality N) arranged columnar in $\mathbb{R}^{n \times N}$, or set of generators, or extreme directions, or matrix variable
G	Gram matrix $X^{\mathrm{T}} X$ (985)
r	affine dimension
$\mathbf{k}$	number of conically independent generators
$\Bbbk$	raw-data domain of Magnetic Resonance Imaging machine as in $\Bbbk$-*space*
n	Euclidean (ambient spatial) dimension of list $X \in \mathbb{R}^{n \times N}$, or integer
N	cardinality of list $X \in \mathbb{R}^{n \times N}$, or integer
in	*function* f *in* x means x as argument to f or x *in* $\mathcal{C}$ means element x is a member of set $\mathcal{C}$
on	*function* $f(x)$ *on* $\mathcal{A}$ means $\mathcal{A}$ is $\operatorname{dom} f$ or *relation* $\preceq$ *on* $\mathcal{A}$ means $\mathcal{A}$ is set whose elements are subject to $\preceq$ or *projection of* x *on* $\mathcal{A}$ means $\mathcal{A}$ is body on which projection is made or *operating on vector* identifies argument type to f as "vector"
onto	*function* $f(x)$ *maps onto* $\mathcal{M}$ means f over its domain is a surjection with respect to $\mathcal{M}$
over	*function* $f(x)$ *over* $\mathcal{C}$ means f evaluated at each and every element of set $\mathcal{C}$

one-to-one	injective map or unique correspondence between sets
epi	function epigraph
dom	function domain
$\mathcal{R}f$	function range
$\mathcal{R}(A)$	the subspace: *range of* A (142) or span basis $\mathcal{R}(A)$; $\mathcal{R}(A) \perp \mathcal{N}(A^{\mathrm{T}})$
span	as in span $A = \mathcal{R}(A) = \{Ax \mid x\in\mathbb{R}^n\}$ when A is a matrix
basis $\mathcal{R}(A)$	*overcomplete columnar basis for range of* A or *minimal set constituting generators for vertex-description of* $\mathcal{R}(A)$ or *linearly independent set of vectors spanning* $\mathcal{R}(A)$
$\mathcal{N}(A)$	the subspace: *nullspace of* A (143) `a.k.a` *kernel of* A ; $\mathcal{N}(A) \perp \mathcal{R}(A^{\mathrm{T}})$
$\mathbb{R}^n$	Euclidean n-dimensional real vector space (nonnegative integer n). $\mathbb{R}^{\mathbf{0}} = \mathbf{0}$, $\mathbb{R} = \mathbb{R}^{\mathbf{1}}$ or vector space of unspecified dimension. [243] [419]
$\mathbb{R}^{m\times n}$	Euclidean vector space of m by n dimensional real matrices
$\times$	Cartesian product. $\mathbb{R}^{m\times n-m} \triangleq \mathbb{R}^{m\times(n-m)}$. $\mathcal{K}_1\times\mathcal{K}_2 = \begin{bmatrix}\mathcal{K}_1\\ \mathcal{K}_2\end{bmatrix}$
$\begin{bmatrix}\mathbb{R}^m\\ \mathbb{R}^n\end{bmatrix}$	$\mathbb{R}^m\times\mathbb{R}^n = \mathbb{R}^{m+n}$
$\mathbb{C}^n$ or $\mathbb{C}^{n\times n}$	Euclidean complex vector space of respective dimension n and $n\times n$
$\mathbb{R}^n_+$ or $\mathbb{R}^{n\times n}_+$	nonnegative orthant in Euclidean vector space of respective dimension n and $n\times n$
$\mathbb{R}^n_-$ or $\mathbb{R}^{n\times n}_-$	nonpositive orthant in Euclidean vector space of respective dimension n and $n\times n$
$\mathbb{S}^n$	subspace of real symmetric $n\times n$ matrices; the *symmetric matrix subspace.* $\mathbb{S}^{\mathbf{0}} = \mathbf{0}$, $\mathbb{S} = \mathbb{S}^{\mathbf{1}}$ or symmetric subspace of unspecified dimension.
$\mathbb{S}^{n\perp}$	orthogonal complement of $\mathbb{S}^n$ in $\mathbb{R}^{n\times n}$, the antisymmetric matrices (51)
$\mathbb{S}^n_+$	convex cone comprising all (real) symmetric positive semidefinite $n\times n$ matrices, the *positive semidefinite cone*
int $\mathbb{S}^n_+$	interior of convex cone comprising all (real) symmetric positive semidefinite $n\times n$ matrices; *id est*, positive definite matrices
$\mathbb{S}^n_+(\rho)$	$= \{X\in\mathbb{S}^n_+ \mid \operatorname{rank} X \geq \rho\}$ (260) convex set of all positive semidefinite $n\times n$ symmetric matrices whose rank equals or exceeds ρ
$\mathbb{EDM}^N$	cone of $N\times N$ Euclidean distance matrices in the symmetric hollow subspace
$\sqrt{\mathbb{EDM}^N}$	nonconvex cone of $N\times N$ Euclidean absolute distance matrices in the symmetric hollow subspace (§6.3)

CPU	central processing unit
dB	decibel
DC	direct current (0Hz)
DCT	discrete cosine transform
DFT	discrete Fourier transform
Hz	hertz (cycles per second), MHz means *megahertz,* GHz *gigahertz*
EDM	Euclidean distance matrix
PSD	positive semidefinite
SDP	semidefinite program
SVD	singular value decomposition
SNR	signal to noise ratio
$\mathbb{S}_1^n$	subspace comprising all symmetric $n\times n$ matrices having all zeros in first row and column (2117)
$\mathbb{S}_h^n$	subspace comprising all symmetric hollow $n\times n$ matrices $\big($$\mathbf{0}$ main diagonal$\big)$, the *symmetric hollow subspace* (66)
$\mathbb{S}_h^{n\perp}$	orthogonal complement of $\mathbb{S}_h^n$ in $\mathbb{S}^n$ (67), the set of all diagonal matrices
$\mathbb{S}_c^n$	subspace comprising all geometrically centered symmetric $n\times n$ matrices; *geometric center subspace* $\mathbb{S}_c^N=\{Y\in\mathbb{S}^N \mid Y\mathbf{1}=\mathbf{0}\}$ (2113)
$\mathbb{S}_c^{n\perp}$	orthogonal complement of $\mathbb{S}_c^n$ in $\mathbb{S}^n$ (2115)
$\mathbb{R}_c^{m\times n}$	subspace comprising all geometrically centered $m\times n$ matrices
$X^\perp$	basis $\mathcal{N}(X^{\mathrm{T}})$ (§2.13.9, §E.3.4)
$x^\perp$	$\mathcal{N}(x^{\mathrm{T}})$; $\{y\in\mathbb{R}^n \mid x^{\mathrm{T}}y=0\}$ (§2.13.10.1.1)
$\mathcal{R}(P)^\perp$	$\mathcal{N}(P^{\mathrm{T}})$; orthogonal complement of $\mathcal{R}(P)$ $\big($fundamental subspace relations (137)$\big)$
$\mathcal{N}(P)^\perp$	$\mathcal{R}(P^{\mathrm{T}})$
$\mathcal{R}^\perp$	$=\{y\in\mathbb{R}^n \mid \langle x,y\rangle=0\ \forall\, x\in\mathcal{R}\}$ (372). *Orthogonal complement of* $\mathcal{R}$ *in* $\mathbb{R}^n$ when $\mathcal{R}$ is a subspace
$\mathcal{K}^\perp$	normal cone (448)
$\mathcal{A}^\perp$	normal cone to affine subset $\mathcal{A}$ (§3.1.2.2.2)
$\mathcal{K}$	cone
$\mathcal{K}^*$	dual cone $-\mathcal{K}^\circ$

$\mathcal{K}^\circ$	polar cone $-\mathcal{K}^*$, or angular *degree* as in 360°
$\mathcal{K}_{\mathcal{M}+}$	monotone nonnegative cone
$\mathcal{K}_{\mathcal{M}}$	monotone cone
$\mathcal{K}_\lambda$	spectral cone
$\mathcal{K}^*_{\lambda\delta}$	cone of majorization
$\mathcal{H}$	halfspace
$\mathcal{H}_-$	halfspace described using an outward-normal (106) to the hyperplane partially bounding it
$\mathcal{H}_+$	halfspace described using an inward-normal (107) to the hyperplane partially bounding it
$\partial\mathcal{H}$	hyperplane; *id est*, partial boundary of halfspace
$\underline{\partial\mathcal{H}}$	supporting hyperplane
$\underline{\partial\mathcal{H}}_-$	a supporting hyperplane having outward-normal with respect to set it supports
$\underline{\partial\mathcal{H}}_+$	a supporting hyperplane having inward-normal with respect to set it supports
$\underline{d}$	vector of distance-square
$\underline{d_{ij}}$	lower bound on distance-square d_{ij}
$\overline{d_{ij}}$	upper bound on distance-square d_{ij}
$\overline{\mathrm{AB}}$	closed line segment between points A and B
AB	matrix multiplication of A and B
$\overline{\mathcal{C}}$	*closure of set* $\mathcal{C}$
decomposition	*orthonormal* (2016, p.613), *biorthogonal* (1992, p.607)
expansion	*orthogonal* (2026, p.615), *biorthogonal* (402, p.164)
vector	*column vector* in $\mathbb{R}^n$
entry	*scalar element or real variable constituting a vector or matrix*
cubix	member of $\mathbb{R}^{M\times N\times L}$
quartix	member of $\mathbb{R}^{M\times N\times L\times K}$
feasible set	most simply, *the set of all variable values satisfying all constraints of an optimization problem*
solution set	most simply, *the set of all optimal solutions to an optimization problem*; a subset of the feasible set and not necessarily a single point

optimal	as in *optimal solution,* means a solution to an optimization problem. An optimal solution is not necessarily unique, but there is no better solution. optimal ⇒ feasible
feasible	as in *feasible solution,* means satisfies the (“subject to”) constraints of an optimization problem, may or may not be optimal
same	as in *same problem,* means optimal solution set for one problem is identical to optimal solution set of another (without transformation)
equivalent	as in *equivalent problem,* means optimal solution to one problem can be derived from optimal solution to another via suitable transformation
convex	as in *convex problem,* essentially means a convex objective function optimized over a convex set (§4)
objective	the three objectives of Optimization are *minimize, maximize,* and *find*
program	*Semidefinite program* is any convex minimization, maximization, or feasibility problem constraining a variable to a subset of a positive semidefinite cone. *Prototypical semidefinite program* conventionally means: a semidefinite program having linear objective, affine equality constraints, but no inequality constraints except for cone membership. (§4.1.1) *Linear program* is any feasibility problem, or minimization or maximization of a linear objective, constraining the variable to some polyhedron (§2.13.1.0.3)
natural order	with reference to stacking columns in a vectorization means *a vector made from superposing column* 1 *on top of column* 2 *then superposing the result on column* 3 and so on; as in a vector made from entries of the main diagonal $\delta(A)$ means *taken from left to right and top to bottom*
partial order	relation $\preceq$ is a partial order, on a set, if it possesses reflexivity, antisymmetry, and transitivity (§2.7.2.2)
operator	mapping to a vector space (a multidimensional function)
projector	short for *projection operator*; not necessarily minimum-distance or represented by a matrix
sparsity	ratio of number of nonzero entries to matrix-dimension product
tight	with reference to a bound means *a bound that can be met,* with reference to an inequality means *equality is achievable*
$\ddot{v}$	coefficient vector for two spectral factors, Figure **104** `level` 2
$\dddot{v}$	coefficient vector corresponding to four spectral factors, Figure **104** `level` 3
$\ddddot{v}$	vector containing numerator or denominator coefficients of eight spectral factors; `level` 4 in a bifurcation tree like Figure **104**
g'	first derivative of possibly multidimensional function with respect to real argument or primed variable closely related to unprimed

g''	second derivative with respect to real argument
$\overset{\rightarrow Y}{dg}$	first directional derivative of possibly multidimensional function g in direction $Y\in\mathbb{R}^{K\times L}$ (maintains dimensions of g)
$\overset{\rightarrow Y}{dg^2}$	second directional derivative of g in direction Y
∇	*gradient* from calculus, ∇f is shorthand for $\nabla_x f(x)$. $\nabla f(y)$ means $\nabla_y f(y)$ or *gradient* $\nabla_x f(y)$ *of* $f(x)$ *with respect to* x *evaluated at* y
∇^2	*second-order gradient*
Δ	distance scalar (Figure **28**), or first-order difference matrix (904), or infinitesimal difference operator (§D.1.4)
$\triangle_{ijk}$	triangle made by vertices i , j , and k
I	Roman numeral one or capital i
I	Identity operator or matrix $I=\delta^2(I)$, $\delta(I)=\mathbf{1}$. Variant: $I_m\triangleq I\in\mathbb{S}^m$
$\mathcal{I}$	*index set,* a discrete set of indices
$\emptyset$	*empty set,* an implicit member of every set
0	real zero
$\mathbf{0}$	*origin* or vector or matrix of zeros
O	*sort-index matrix*
O	*order of magnitude* information required, or *computational intensity*: O(N) is first order, O(N^2) is second, and so on
1	real one
$\mathbf{1}$	vector of ones. Variant: $\mathbf{1}_m\triangleq\mathbf{1}\in\mathbb{R}^m$
e_i	vector whose i^{th} entry is 1 (otherwise 0), i^{th} *member of the standard basis for* $\mathbb{R}^n$ (60)
max	*maximum* [215, §0.1.1] or *largest element of a totally ordered set*
$\underset{x}{\text{maximize}}$	*find maximum of objective function w.r.t independent variables* x . Subscript $x\leftarrow x\in\mathcal{C}$ may hold implicit constraints if context clear; *e.g*, semidefiniteness
arg	*argument* of operator or function, or *variable of optimization*
$\sup\mathcal{X}$	*supremum* of totally ordered set $\mathcal{X}$, *least upper bound,* may or may not belong to set [215, §0.1.1]; *e.g*, range $\mathcal{X}$ of real function

$\arg\sup f(x)$	*argument* x *at supremum of function* f ; not necessarily unique or a member of function domain
subject to	specifies constraints of an optimization problem; generally, inequalities and affine equalities. *Subject to* implies: anything not an independent variable is constant, an assignment, or substitution
min	*minimum* [215, §0.1.1] or *smallest element of a totally ordered set*
$\underset{x}{\text{minimize}}$	*find objective function minimum w.r.t independent variables* x . Subscript $x \leftarrow x\in\mathcal{C}$ may hold implicit constraints if context clear; *e.g*, semidefiniteness
$\underset{x}{\text{find}}$	*find any feasible solution, specified by the* (*"subject to"*) *constraints, w.r.t independent variables* x . Subscript $x \leftarrow x\in\mathcal{C}$ may hold implicit constraints if context clear; *e.g*, semidefiniteness. "find" denotes a *feasibility problem*; it is the third objective of Optimization
$\inf\mathcal{X}$	*infimum* of totally ordered set $\mathcal{X}$, *greatest lower bound,* may or may not belong to set [215, §0.1.1]; *e.g*, range $\mathcal{X}$ of real function
$\arg\inf f(x)$	*argument* x *at infimum of function* f ; not necessarily unique or a member of function domain
iff	*if and only if* , *necessary and sufficient*; also the meaning indiscriminately attached to appearance of the word "if" in the statement of a mathematical definition, [139, p.106] [274, p.4] an esoteric practice worthy of abolition because of ambiguity thus conferred
rel	relative
int	interior
lim	limit
sgn	signum function or *sign*
round	round to nearest integer
mod	modulus function
tr	matrix trace
rank	as in $\operatorname{rank} A$, *rank of matrix* A ; $\dim\mathcal{R}(A)$
dim	dimension, $\dim\mathbb{R}^n = n$, $\dim(x\in\mathbb{R}^n) = n$, $\dim\mathcal{R}(x\in\mathbb{R}^n) = 1$, $\dim\mathcal{R}(A\in\mathbb{R}^{m\times n}) = \operatorname{rank}(A)$
aff	affine hull
dim aff	affine dimension

card	cardinality, *number of nonzero entries* $\operatorname{card} x \triangleq \lVert x\rVert_0$ or N *is cardinality of list* $X\in\mathbb{R}^{n\times N}$ (p.278)
conv	convex hull (§2.3.2)
cone	conic hull (§2.3.3)
cenv	convex envelope (§7.2.2.1)
content	of high-dimensional bounded polyhedron, volume in 3 dimensions, area in 2, and so on
cof	matrix of cofactors corresponding to matrix argument
dist	absolute distance between point or set arguments; *e.g*, $\operatorname{dist}(x\,,\mathcal{B})$
vec	columnar vectorization of $m\times n$ matrix, Euclidean dimension mn (37)
svec	columnar vectorization of symmetric $n\times n$ matrix, Euclidean dimension $n(n+1)/2$ (56)
dvec	columnar vectorization of symmetric hollow $n\times n$ matrix, Euclidean dimension $n(n-1)/2$ (73)
$\measuredangle(x\,,y)$	*angle* between vectors x and y, or *dihedral angle* between affine subsets
$\succeq$	generalized inequality; *e.g*, $A\succeq 0$ means: • *vector or matrix* A *must be expressible in a biorthogonal expansion having nonnegative coordinates with respect to extreme directions of some implicit pointed closed convex cone* $\mathcal{K}$ (§2.13.2.0.1, §2.13.7.1.1), • **or** *comparison to the origin with respect to some implicit pointed closed convex cone* (2.7.2.2), • **or** (when $\mathcal{K}=\mathbb{S}_+^n$) *matrix* A *belongs to the positive semidefinite cone of symmetric matrices* (nonnegative eigenvalues, §2.9.0.1), • **or** (when $\mathcal{K}=\mathbb{R}_+^n$) *vector* A *belongs to the nonnegative orthant* (each vector entry is nonnegative, §2.3.1.1)
$\underset{\mathcal{K}}{\succeq}$	as in $x \underset{\mathcal{K}}{\succeq} z$ means $x-z\in\mathcal{K}$ (182)
$\succ$	strict generalized inequality, membership to cone interior; $A\succ 0$ means: • *vector or matrix* A *must be expressible in a biorthogonal expansion having positive coordinates with respect to extreme directions of some implicit pointed closed convex cone* $\mathcal{K}$ (§2.13.2.0.1, §2.13.7.1.1), • **or** *comparison to the origin with respect to the interior of some implicit pointed closed convex cone* (2.7.2.2), • **or** (when $\mathcal{K}=\mathbb{S}_+^n$) *matrix* A *belongs to the interior of the positive semidefinite cone of symmetric matrices* (positive eigenvalues, §2.9.0.1), • **or** (when $\mathcal{K}=\mathbb{R}_+^n$) *vector* A *belongs to the interior of the nonnegative orthant* (each vector entry is positive, §2.3.1.1)

$\nsucc$	not positive definite		
$\geq$	scalar inequality, *greater than or equal to*; comparison of scalars, or entrywise comparison of vectors or matrices with respect to $\mathbb{R}_+$		
nonnegative	for $\alpha\in\mathbb{R}^n$, $\alpha\succeq 0$; *id est*, nonnegative entries when with respect to nonnegative orthant, represents vector on boundary of or interior to pointed closed convex cone $\mathcal{K}$		
$>$	*greater than*		
positive	for $\alpha\in\mathbb{R}^n$, $\alpha\succ 0$; *id est*, positive (nonzero) entries when with respect to nonnegative orthant, no vector on boundary of pointed closed convex cone $\mathcal{K}$		
$\lfloor\ \rfloor$	floor function, $\lfloor x\rfloor$ is greatest integer not exceeding x		
$\lvert\ \rvert$	entrywise absolute value of scalars, vectors, and matrices		
log	natural (or Napierian) logarithm		
det	matrix determinant		
$\lVert x\rVert$	$= \sqrt{\sum\limits_{j=1}^{n}\lvert x_j\rvert^2}$	*Euclidean norm* or vector 2-norm $\lVert x\rVert_2$	(§3.2)
$\lVert x\rVert_2^2$	$= x^{\mathrm T}x = \langle x\,,\,x\rangle$	*Euclidean norm square*	(§3.1.2.1)
$\lVert x\rVert_\ell$	$= \sqrt[\ell]{\sum\limits_{j=1}^{n}\lvert x_j\rvert^\ell}$	vector *ℓ-norm* for $\ell\geq 1$ (convex)	
	$\triangleq \sum\limits_{j=1}^{n}\lvert x_j\rvert^\ell$	vector *ℓ-norm* for $0\leq\ell<1$	(violates §3.2 *no.*3)
$\lVert x\rVert_0$	$= \mathbf{1}^{\mathrm T}\lvert x\rvert^0 \quad (0^0\triangleq 0)$	"0-norm" or *cardinality of vector x* (card x)	(§4.5.1)
$\lVert x\rVert_1$	$= \mathbf{1}^{\mathrm T}\lvert x\rvert$	1-norm, *dual infinity-norm*	(§3.2)
$\lVert x\rVert_\infty$	$= \max\{\lvert x_j\rvert\ \forall j\}$	*infinity-norm*	(§3.2)
$\lVert x\rVert_{\substack{n\\k}}$	$= \sum\limits_{i=1}^{k}\pi(\lvert x\rvert)_i$	*k-largest norm*	(§3.2.2.1)
$\lVert X\rVert_2$	$= \sup\limits_{\lVert a\rVert=1}\lVert Xa\rVert_2 = \sigma_1 = \sqrt{\lambda(X^{\mathrm T}X)_1}$	matrix 2-norm or *spectral norm, largest singular value* [174, p.56]. For x a vector: $\lVert\delta(x)\rVert_2 = \lVert x\rVert_\infty$.	(591)

$$\lVert Xa\rVert_2 \leq \lVert X\rVert_2\lVert a\rVert_2 \tag{2209}$$

$\lVert X\rVert_2^*$	$= \mathbf{1}^{\mathrm T}\sigma(X)$	*nuclear norm, dual spectral norm*	(§C.2)
$\lVert X\rVert$	$= \sqrt{\sum\limits_{i,j}X_{ij}^2}$	*Frobenius' matrix norm* $\lVert X\rVert_{\mathrm F}$	(§2.2.1)