

Appendix F

Notation and a few definitions

b	(italic $abcdefghijklmnopqrstuvwxy$) column vector, scalar, logical condition
b_i	i^{th} entry of vector $b = [b_i, i = 1 \dots n]$ or i^{th} b vector from a set or list $\{b_j, j = 1 \dots n\}$ or i^{th} iterate of vector b
$b_{i:j}$	or $b(i:j)$, truncated vector comprising i^{th} through j^{th} entry of vector b
$b_k(i:j)$	or $b_{i:j,k}$, truncated vector comprising i^{th} through j^{th} entry of vector b_k
b^T	vector transpose
b^H	Hermitian (complex conjugate) transpose b^{*T}
A^T	Matrix transpose. Regarding A as a linear operator, A^T is its adjoint.
A^{-2T}	matrix transpose of squared inverse
A^{T_1}	first of various transpositions of a cubix or quartix A (p.666, p.671)
A	matrix, scalar, or logical condition (italic $ABCDEFGHIJKLMNPOQRSTUVWXYZ$)
<i>skinny</i>	a skinny matrix; meaning, more rows than columns: $\begin{bmatrix} \\ \\ \end{bmatrix}$. When there are more equations than unknowns, we say that the system $Ax = b$ is overdetermined. [155, §5.3]

<i>fat</i>	a fat matrix; meaning, more columns than rows: $\left[\begin{array}{c} \\ \end{array} \right]$ <i>underdetermined</i>
\mathcal{A}	some set (calligraphic $ABCDEFGHIJKLMNOPQRSTUVWXYZ$)
$\mathcal{F}(\mathcal{C} \ni A)$	smallest face (162) that contains element A of set \mathcal{C}
$\mathcal{G}(\mathcal{K})$	generators (§2.13.4.2.1) of set \mathcal{K} ; any collection of points and directions whose hull constructs \mathcal{K}
\mathcal{L}_ν^ν	level set (533)
\mathcal{L}_ν	sublevel set (537)
\mathcal{L}^ν	superlevel set (620)
A^{-1}	inverse of matrix A
A^\dagger	Moore-Penrose pseudoinverse of matrix A
$\sqrt{\quad}$	positive square root
$\sqrt[\ell]{\quad}$	positive ℓ^{th} root
$A^{1/2}$ and \sqrt{A}	$A^{1/2}$ is any matrix such that $A^{1/2}A^{1/2} = A$. For $A \in \mathbb{S}_+^n$, $\sqrt{A} \in \mathbb{S}_+^n$ is unique and $\sqrt{A}\sqrt{A} = A$. [53, §1.2] (§A.5.1.3)
$\sqrt[D]{\quad}$	$= [\sqrt{d_{ij}}]$. (1323) <i>Hadamard positive square root</i> : $D = \sqrt[D]{\quad} \circ \sqrt[D]{\quad}$
\mathfrak{A}	Euler Fraktur $\mathfrak{A} \mathfrak{B} \mathfrak{C} \mathfrak{D} \mathfrak{E} \mathfrak{F} \mathfrak{G} \mathfrak{H} \mathfrak{I} \mathfrak{J} \mathfrak{K} \mathfrak{L} \mathfrak{M} \mathfrak{N} \mathfrak{O} \mathfrak{P} \mathfrak{Q} \mathfrak{R} \mathfrak{S} \mathfrak{T} \mathfrak{U} \mathfrak{V} \mathfrak{W} \mathfrak{X} \mathfrak{Y} \mathfrak{Z}$
\mathcal{L}	Lagrangian (485)
\mathcal{E}	member of elliptope \mathcal{E}_t (1073) parametrized by scalar t
\mathcal{E}	elliptope (1052)
E	elementary matrix
E_{ij}	member of standard orthonormal basis for symmetric (59) or symmetric hollow (75) matrices

A_{ij} or $A(i, j)$, ij^{th} entry of matrix $A = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{matrix} & \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{matrix}$
 or rank-one matrix $a_i a_j^{\text{T}}$ (§4.7)

$A(i, j)$ A is a function of i and j

A_i i^{th} matrix from a set or i^{th} principal submatrix or i^{th} iterate of A

$A(i, :)$ i^{th} row of matrix A

$A(:, j)$ j^{th} column of matrix A [155, §1.1.8]

$A_{i:j, k:\ell}$ or $A(i:j, k:\ell)$, submatrix taken from i^{th} through j^{th} row and k^{th} through ℓ^{th} column

e.g. *exempli gratia*, from the Latin meaning *for sake of example*

videlicet from the Latin meaning *it is permitted to see*

no. *number*, from the Latin *numero*

a.i. affinely independent (§2.4.2.3)

c.i. conically independent (§2.10)

l.i. linearly independent

w.r.t *with respect to*

a.k.a *also known as*

re real part

im imaginary part

i or j $\sqrt{-1}$

$\subset \supset \cap \cup$ standard set theory, *subset, superset, intersection, union*

\in membership, *element belongs to*, or *element is a member of*

\ni membership, *contains* as in $\mathcal{C} \ni y$ (\mathcal{C} contains element y)

- \ni *such that*
- \exists *there exists*
- \therefore *therefore*
- \forall *for all, or over all*
- \propto *proportional to*
- ∞ *infinity*
- \equiv *equivalent to*
- \triangleq *defined equal to, equal by definition*
- \approx *approximately equal to*
- \simeq *isomorphic to or with*
- \cong *congruent to or with*
- Hadamard quotient as in, for $x, y \in \mathbb{R}^n$, $\frac{x}{y} \triangleq [x_i/y_i, i=1 \dots n] \in \mathbb{R}^n$
- Hadamard product of matrices: $x \circ y \triangleq [x_i y_i, i=1 \dots n] \in \mathbb{R}^n$
- ⊗ Kronecker product of matrices (§D.1.2.1)
- ⊕ vector sum of sets $\mathcal{X} = \mathcal{Y} \oplus \mathcal{Z}$ where every element $x \in \mathcal{X}$ has unique expression $x = y + z$ where $y \in \mathcal{Y}$ and $z \in \mathcal{Z}$; [301, p.19] then summands are algebraic complements. $\mathcal{X} = \mathcal{Y} \oplus \mathcal{Z} \Rightarrow \mathcal{X} = \mathcal{Y} + \mathcal{Z}$. Now assume \mathcal{Y} and \mathcal{Z} are nontrivial subspaces. $\mathcal{X} = \mathcal{Y} + \mathcal{Z} \Rightarrow \mathcal{X} = \mathcal{Y} \oplus \mathcal{Z} \Leftrightarrow \mathcal{Y} \cap \mathcal{Z} = \mathbf{0}$ [302, §1.2] [106, §5.8]. Each element from a vector sum (+) of subspaces has unique expression (⊕) when a basis from each subspace is linearly independent of bases from all the other subspaces.
- ⊖ likewise, the vector difference of sets

- \boxplus orthogonal vector sum of sets $\mathcal{X} = \mathcal{Y} \boxplus \mathcal{Z}$ where every element $x \in \mathcal{X}$ has unique orthogonal expression $x = y + z$ where $y \in \mathcal{Y}$, $z \in \mathcal{Z}$, and $y \perp z$. [324, p.51] $\mathcal{X} = \mathcal{Y} \boxplus \mathcal{Z} \Rightarrow \mathcal{X} = \mathcal{Y} + \mathcal{Z}$. If $\mathcal{Z} \subseteq \mathcal{Y}^\perp$ then $\mathcal{X} = \mathcal{Y} \boxplus \mathcal{Z} \Leftrightarrow \mathcal{X} = \mathcal{Y} \oplus \mathcal{Z}$. [106, §5.8] If $\mathcal{Z} = \mathcal{Y}^\perp$ then summands are orthogonal complements.
- \pm *plus or minus* or *plus and minus*
- \perp as in $A \perp B$ meaning A is orthogonal to B (and vice versa), where A and B are sets, vectors, or matrices. When A and B are vectors (or matrices under Frobenius' norm), $A \perp B \Leftrightarrow \langle A, B \rangle = 0 \Leftrightarrow \|A + B\|^2 = \|A\|^2 + \|B\|^2$
- \setminus as in $\setminus \mathcal{A}$ means logical *not* \mathcal{A} , or *relative complement of set* \mathcal{A} ; *id est*, $\setminus \mathcal{A} = \{x \notin \mathcal{A}\}$; e.g., $\mathcal{B} \setminus \mathcal{A} \triangleq \{x \in \mathcal{B} \mid x \notin \mathcal{A}\} \equiv \mathcal{B} \cap \setminus \mathcal{A}$
- \Rightarrow or \Leftarrow sufficient or necessary, *implies*, or *is implied by*; e.g.,
 A is sufficient: $A \Rightarrow B$, A is necessary: $A \Leftarrow B$,
 $A \Rightarrow B \Leftrightarrow \setminus A \Leftarrow \setminus B$, $A \Leftarrow B \Leftrightarrow \setminus A \Rightarrow \setminus B$,
if A *then* B , *if* B *then* A ,
 A *only if* B . B *only if* A .
- \Leftrightarrow *if and only if* (iff) or *corresponds with* or *necessary and sufficient* or *logical equivalence*
- is* as in A *is* B means $A \Rightarrow B$; conventional usage of English language imposed by logicians
- \nRightarrow or \nLeftarrow insufficient or unnecessary, *does not imply*, or *is not implied by*; e.g.,
 $A \nRightarrow B \Leftrightarrow \setminus A \nLeftarrow \setminus B$. $A \nLeftarrow B \Leftrightarrow \setminus A \nRightarrow \setminus B$.
- \leftarrow *is replaced with*; substitution, assignment
- \rightarrow *goes to*, or *approaches*, or *maps to*
- $t \rightarrow 0^+$ t *goes to* 0 *from above*; meaning, *from the positive* [195, p.2]
- \vdots \ddots \dots as in $1 \dots 1$ meaning *continuation*, e.g., a sequence of ones in a row
- \dots as in $i = 1 \dots N$ meaning, i is a sequence of successive integers beginning with 1 and ending with N

:	as in $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ meaning f is a mapping, or sequence of successive integers specified by bounds as in $i:j$ (if $j < i$ then sequence is descending)
$f : \mathcal{M} \rightarrow \mathcal{R}$	meaning f is a mapping from ambient space \mathcal{M} to ambient \mathcal{R} , not necessarily denoting either domain or range
	as in $f(x) x \in \mathcal{C}$ means with the condition(s) or such that or evaluated for, or as in $\{f(x) x \in \mathcal{C}\}$ means evaluated for each and every x belonging to set \mathcal{C}
$g _{x_p}$	expression g evaluated at x_p
A, B	as in, for example, $A, B \in \mathbb{S}^N$ means $A \in \mathbb{S}^N$ and $B \in \mathbb{S}^N$
(A, B)	open interval between A and B in \mathbb{R} , or variable pair perhaps of disparate dimension
$[A, B]$	closed interval or line segment between A and B in \mathbb{R}
()	hierarchal, parenthetical, optional
{ }	curly braces denote a set or list, e.g., $\{Xa a \succeq 0\}$ the set comprising Xa evaluated for each and every $a \succeq 0$ where membership of a to some space is implicit, a union
$\langle \rangle$	angle brackets denote vector inner-product (33) (38)
[]	matrix or vector, or quote insertion, or citation
$[d_{ij}]$	matrix whose ij^{th} entry is d_{ij}
$[x_i]$	vector whose i^{th} entry is x_i
x_p	particular value of x
x_0	particular instance of x , or initial value of a sequence x_i
x_1	first entry of vector x , or first element of a set or list $\{x_i\}$
x_ε	extreme point

x_+	vector x whose negative entries are replaced with 0; $x_+ = \frac{1}{2}(x + x)$ (513) or <i>clipped vector</i> x or <i>nonnegative part of</i> x
x_-	$x_- \triangleq \frac{1}{2}(x - x)$ or <i>nonpositive part of</i> $x = x_+ + x_-$
\check{x}	known data
x^*	optimal value of variable x
x^*	<i>complex conjugate</i> or <i>dual variable</i> or <i>extreme direction of dual cone</i>
f^*	<i>convex conjugate function</i> $f^*(s) = \sup\{\langle s, x \rangle - f(x) \mid x \in \text{dom } f\}$
$P_C x$ or Px	projection of point x on set C , P is operator or idempotent matrix
$P_k x$	projection of point x on set C_k or on range of implicit vector
$\delta(A)$	(§A.1) <i>vector made from the main diagonal of</i> A if A is a matrix; otherwise, <i>diagonal matrix made from vector</i> A
$\delta^2(A)$	$\equiv \delta(\delta(A))$. For vector or diagonal matrix Λ , $\delta^2(\Lambda) = \Lambda$
$\delta(A)^2$	$= \delta(A)\delta(A)$ where A is a vector
$\lambda_i(X)$	i^{th} entry of vector λ is function of X
$\lambda(X)_i$	i^{th} entry of vector-valued function of X
$\lambda(A)$	<i>vector of eigenvalues of matrix</i> A , (1437) typically arranged in nonincreasing order
$\sigma(A)$	<i>vector of singular values of matrix</i> A (always arranged in nonincreasing order), or <i>support function in direction</i> A
Σ	diagonal matrix of singular values, not necessarily square
\sum	sum
$\pi(\gamma)$	nonlinear <i>permutation operator</i> (or <i>presorting function</i>) arranges vector γ into nonincreasing order (§7.1.3)
Ξ	permutation matrix
Π	doublet or permutation operator or matrix

\prod	product
$\psi(Z)$	signum-like <i>step function</i> that returns a scalar for matrix argument (684), it returns a vector for vector argument (1545)
D	symmetric hollow matrix of distance-square, or <i>Euclidean distance matrix</i>
\mathbf{D}	Euclidean distance matrix operator
$\mathbf{D}^T(X)$	adjoint operator
$\mathbf{D}(X)^T$	transpose of $\mathbf{D}(X)$
$\mathbf{D}^{-1}(X)$	inverse operator
$\mathbf{D}(X)^{-1}$	inverse of $\mathbf{D}(X)$
D^*	optimal value of variable D
D^*	dual to variable D
D°	polar variable D
∂	<i>partial derivative</i> or <i>partial differential</i> or <i>matrix of distance-square squared</i> (1363) or <i>boundary</i> of set \mathcal{K} as in $\partial\mathcal{K}$
$\sqrt{d_{ij}}$	(absolute) distance scalar
d_{ij}	distance-square scalar, EDM entry
\mathbf{V}	geometric centering operator, $\mathbf{V}(D) = -VDV^{\frac{1}{2}}$ (972)
$\mathbf{V}_{\mathcal{N}}$	$\mathbf{V}_{\mathcal{N}}(D) = -V_{\mathcal{N}}^T D V_{\mathcal{N}}$ (986)
V	$N \times N$ symmetric elementary, auxiliary, projector, geometric centering matrix, $\mathcal{R}(V) = \mathcal{N}(\mathbf{1}^T)$, $\mathcal{N}(V) = \mathcal{R}(\mathbf{1})$, $V^2 = V$ (§B.4.1)
$V_{\mathcal{N}}$	$N \times N - 1$ Schoenberg auxiliary matrix, $\mathcal{R}(V_{\mathcal{N}}) = \mathcal{N}(\mathbf{1}^T)$, $\mathcal{N}(V_{\mathcal{N}}^T) = \mathcal{R}(\mathbf{1})$ (§B.4.2)
$V_{\mathcal{X}}$	$V_{\mathcal{X}} V_{\mathcal{X}}^T \equiv V^T X^T X V$ (1166)

X	point list ((76) having cardinality N) arranged columnar in $\mathbb{R}^{n \times N}$, or set of generators, or extreme directions, or matrix variable
G	Gram matrix $X^T X$
r	affine dimension
\mathbf{k}	number of conically independent generators or raw-data domain of Magnetic Resonance Imaging machine as in \mathbf{k} -space
n	Euclidean (ambient spatial) dimension of list $X \in \mathbb{R}^{n \times N}$, or integer
N	cardinality of list $X \in \mathbb{R}^{n \times N}$, or integer
<i>on</i>	<i>function $f(x)$ on \mathcal{A}</i> means \mathcal{A} is $\text{dom } f$, or <i>projection of x on \mathcal{A}</i> means \mathcal{A} is Euclidean body on which projection of x is made
<i>onto</i>	<i>function $f(x)$ maps onto \mathcal{M}</i> means f over its domain is a surjection with respect to \mathcal{M}
<i>one-to-one</i>	injective map or unique correspondence between sets
epi	function epigraph
dom	function domain
$\mathcal{R}f$	function range
$\mathcal{R}(A)$	the subspace: <i>range of A</i> , or span basis $\mathcal{R}(A)$; $\mathcal{R}(A) \perp \mathcal{N}(A^T)$
span	as in $\text{span } A = \mathcal{R}(A) = \{Ax \mid x \in \mathbb{R}^n\}$ when A is a matrix
basis $\mathcal{R}(A)$	<i>columnar basis for range of A</i> , or <i>a minimal set constituting generators for the vertex-description of $\mathcal{R}(A)$</i> , or <i>a linearly independent set of vectors spanning $\mathcal{R}(A)$</i>
$\mathcal{N}(A)$	the subspace: <i>nullspace of A</i> ; $\mathcal{N}(A) \perp \mathcal{R}(A^T)$
\mathbb{R}^n	Euclidean n -dimensional real vector space (nonnegative integer n). $\mathbb{R}^0 = \mathbf{0}$, $\mathbb{R} = \mathbb{R}^1$ or vector space of unspecified dimension.
$\mathbb{R}^{m \times n}$	Euclidean vector space of m by n dimensional real matrices
\times	Cartesian product. $\mathbb{R}^{m \times n - m} \triangleq \mathbb{R}^{m \times (n - m)}$

$\begin{bmatrix} \mathbb{R}^m \\ \mathbb{R}^n \end{bmatrix}$	$\mathbb{R}^m \times \mathbb{R}^n = \mathbb{R}^{m+n}$
\mathbb{C}^n or $\mathbb{C}^{n \times n}$	Euclidean complex vector space of respective dimension n and $n \times n$
\mathbb{R}_+^n or $\mathbb{R}_+^{n \times n}$	nonnegative orthant in Euclidean vector space of respective dimension n and $n \times n$
\mathbb{R}_-^n or $\mathbb{R}_-^{n \times n}$	nonpositive orthant in Euclidean vector space of respective dimension n and $n \times n$
\mathbb{S}^n	subspace of real symmetric $n \times n$ matrices; the <i>symmetric matrix subspace</i> . $\mathbb{S} = \mathbb{S}^1$ or symmetric subspace of unspecified dimension.
$\mathbb{S}^{n\perp}$	orthogonal complement of \mathbb{S}^n in $\mathbb{R}^{n \times n}$, the antisymmetric matrices
\mathbb{S}_+^n	convex cone comprising all (real) symmetric positive semidefinite $n \times n$ matrices, the <i>positive semidefinite cone</i>
$\text{int } \mathbb{S}_+^n$	interior of convex cone comprising all (real) symmetric positive semidefinite $n \times n$ matrices; <i>id est</i> , positive definite matrices
$\mathbb{S}_+^n(\rho)$	$= \{X \in \mathbb{S}_+^n \mid \text{rank } X \geq \rho\}$ (240) convex set of all positive semidefinite $n \times n$ symmetric matrices whose rank equals or exceeds ρ
EDM^N	cone of $N \times N$ Euclidean distance matrices in the symmetric hollow subspace
$\sqrt{\text{EDM}^N}$	nonconvex cone of $N \times N$ Euclidean absolute distance matrices in the symmetric hollow subspace (§6.3)
PSD	positive semidefinite
SDP	semidefinite program
SVD	singular value decomposition
SNR	signal to noise ratio
dB	decibel
EDM	Euclidean distance matrix

\mathbb{S}_1^n	subspace comprising all symmetric $n \times n$ matrices having all zeros in first row and column (1961)
\mathbb{S}_h^n	subspace comprising all symmetric hollow $n \times n$ matrices ($\mathbf{0}$ main diagonal), the <i>symmetric hollow subspace</i> (66)
$\mathbb{S}_h^{n\perp}$	orthogonal complement of \mathbb{S}_h^n in \mathbb{S}^n (67), the set of all diagonal matrices
\mathbb{S}_c^n	subspace comprising all geometrically centered symmetric $n \times n$ matrices; <i>geometric center subspace</i> $\mathbb{S}_c^N = \{Y \in \mathbb{S}^N \mid Y\mathbf{1} = \mathbf{0}\}$ (1957)
$\mathbb{S}_c^{n\perp}$	orthogonal complement of \mathbb{S}_c^n in \mathbb{S}^n (1959)
$\mathbb{R}_c^{m \times n}$	subspace comprising all geometrically centered $m \times n$ matrices
X^\perp	basis $\mathcal{N}(X^T)$
x^\perp	$\mathcal{N}(x^T)$, $\{y \in \mathbb{R}^n \mid x^T y = 0\}$
$\mathcal{R}(P)^\perp$	$\mathcal{N}(P^T)$
\mathcal{R}^\perp	$= \{y \in \mathbb{R}^n \mid \langle x, y \rangle = 0 \forall x \in \mathcal{R}\}$ (351); for \mathcal{R} a subspace, <i>orthogonal complement</i> of \mathcal{R} in \mathbb{R}^n
\mathcal{K}^\perp	normal cone (2046)
\mathcal{K}	<i>cone</i>
\mathcal{K}^*	<i>dual cone</i>
\mathcal{K}°	<i>polar cone</i> ; $\mathcal{K}^* = -\mathcal{K}^\circ$
$\mathcal{K}_{\mathcal{M}+}$	monotone nonnegative cone
$\mathcal{K}_{\mathcal{M}}$	monotone cone
\mathcal{K}_λ	spectral cone
$\mathcal{K}_{\lambda\delta}^*$	cone of majorization
\mathcal{H}	halfspace
\mathcal{H}_-	halfspace described using an outward-normal (105) to the hyperplane partially bounding it

\mathcal{H}_+	halfspace described using an inward-normal (106) to the hyperplane partially bounding it
$\partial\mathcal{H}$	hyperplane; <i>id est</i> , partial boundary of halfspace
$\underline{\partial\mathcal{H}}$	supporting hyperplane
$\overline{\partial\mathcal{H}}_-$	a supporting hyperplane having outward-normal with respect to set it supports
$\overline{\partial\mathcal{H}}_+$	a supporting hyperplane having inward-normal with respect to set it supports
\underline{d}	vector of distance-square
\underline{d}_{ij}	lower bound on distance-square d_{ij}
\overline{d}_{ij}	upper bound on distance-square d_{ij}
\overline{AB}	closed line segment between points A and B
AB	matrix multiplication of A and B
\overline{C}	<i>closure of set C</i>
<i>decomposition</i>	<i>orthonormal (1872) page 698, biorthogonal (1849) page 692</i>
<i>expansion</i>	<i>orthogonal (1882) page 701, biorthogonal (381) page 183</i>
<i>vector</i>	<i>column vector in \mathbb{R}^n</i>
<i>entry</i>	<i>scalar element or real variable constituting a vector or matrix</i>
<i>cubic</i>	<i>member of $\mathbb{R}^{M \times N \times L}$</i>
<i>quartic</i>	<i>member of $\mathbb{R}^{M \times N \times L \times K}$</i>
<i>feasible set</i>	<i>most simply, the set of all variable values satisfying all constraints of an optimization problem</i>
<i>solution set</i>	<i>most simply, the set of all optimal solutions to an optimization problem; a subset of the feasible set and not necessarily a single point</i>
<i>optimal solution</i>	<i>solution to an optimization problem</i>

<i>feasible solution</i>	satisfies (“subject to”) the constraints of an optimization problem, may or may not be optimal
<i>natural order</i>	with reference to stacking columns in a vectorization means <i>a vector made from superposing column 1 on top of column 2 then superposing the result on column 3</i> and so on; as in a vector made from entries of the main diagonal $\delta(A)$ means <i>taken from left to right and top to bottom</i>
<i>operator</i>	mapping to a vector space (a multidimensional function)
<i>tight</i>	with reference to a bound means <i>a bound that can be met</i> , with reference to an inequality means <i>equality is achievable</i>
g'	first derivative of possibly multidimensional function with respect to real argument
g''	second derivative with respect to real argument
$\xrightarrow{Y} dg$	first directional derivative of possibly multidimensional function g in direction $Y \in \mathbb{R}^{K \times L}$ (maintains dimensions of g)
$\xrightarrow{Y} dg^2$	second directional derivative of g in direction Y
∇	<i>gradient</i> from calculus, ∇f is shorthand for $\nabla_x f(x)$. $\nabla f(y)$ means $\nabla_y f(y)$ or <i>gradient $\nabla_x f(y)$ of $f(x)$ with respect to x evaluated at y</i>
∇^2	<i>second-order gradient</i>
Δ	distance scalar (Figure 25), or first-order difference matrix (827), or infinitesimal difference operator (§D.1.4)
Δ_{ijk}	triangle made by vertices i , j , and k
I	Roman numeral one
I	identity matrix
\mathcal{I}	<i>index set</i> , a discrete set of indices
\emptyset	<i>empty set</i> , an implicit member of every set
0	real zero

$\mathbf{0}$	<i>origin</i> or vector or matrix of zeros
O	<i>sort-index matrix</i>
O	<i>order of magnitude</i> information required, or <i>computational intensity</i> : $O(N)$ is first order, $O(N^2)$ is second, and so on
1	real one
$\mathbf{1}$	vector of ones
e_i	vector whose i^{th} entry is 1 (otherwise 0), i^{th} member of the standard basis for \mathbb{R}^n (60)
max	<i>maximum</i> [195, §0.1.1] or <i>largest element of a totally ordered set</i>
maximize x	<i>find the maximum of a function with respect to variable x</i>
arg	<i>argument</i> of operator or function, or <i>variable of optimization</i>
sup \mathcal{X}	<i>supremum</i> of totally ordered set \mathcal{X} , <i>least upper bound</i> , may or may not belong to set [195, §0.1.1]
arg sup $f(x)$	<i>argument x at supremum of function f</i> ; not necessarily unique or a member of function domain
subject to	specifies constraints of an optimization problem
min	<i>minimum</i> [195, §0.1.1] or <i>smallest element of a totally ordered set</i>
minimize x	<i>find the function minimum with respect to variable x</i>
inf \mathcal{X}	<i>infimum</i> of totally ordered set \mathcal{X} , <i>greatest lower bound</i> , may or may not belong to set [195, §0.1.1]
arg inf $f(x)$	<i>argument x at infimum of function f</i> ; not necessarily unique or a member of function domain
iff	<i>if and only if, necessary and sufficient</i> ; also the meaning indiscriminately attached to appearance of the word “if” in the statement of a mathematical definition, [125, p.106] [252, p.4] an esoteric practice worthy of abolition because of ambiguity thus conferred

rel	relative
int	interior
lim	limit
sgn	signum function or <i>sign</i>
round	round to nearest integer
mod	modulus function
tr	matrix trace
rank	as in $\text{rank } A$, <i>rank of matrix</i> A ; $\dim \mathcal{R}(A)$
dim	dimension, $\dim \mathbb{R}^n = n$, $\dim(x \in \mathbb{R}^n) = n$, $\dim \mathcal{R}(x \in \mathbb{R}^n) = 1$, $\dim \mathcal{R}(A \in \mathbb{R}^{m \times n}) = \text{rank}(A)$
aff	affine hull
dim aff	affine dimension
card	cardinality, <i>number of nonzero entries</i> $\text{card } x \triangleq \ x\ _0$ or N is <i>cardinality of list</i> $X \in \mathbb{R}^{n \times N}$ (p.315)
conv	convex hull (§2.3.2)
cone	conic hull (§2.3.3)
cenv	convex envelope (§7.2.2.1)
content	of high-dimensional bounded polyhedron, volume in 3 dimensions, area in 2, and so on
cof	matrix of cofactors corresponding to matrix argument
dist	distance between point or set arguments; <i>e.g.</i> , $\text{dist}(x, \mathcal{B})$
vec	columnar vectorization of $m \times n$ matrix, Euclidean dimension mn (37)
svec	columnar vectorization of symmetric $n \times n$ matrix, Euclidean dimension $n(n+1)/2$ (56)

dvec	columnar vectorization of symmetric hollow $n \times n$ matrix, Euclidean dimension $n(n-1)/2$ (73)
$\sphericalangle(x, y)$	angle between vectors x and y , or dihedral angle between affine subsets
\succeq	generalized inequality; e.g., $A \succeq 0$ means: vector or matrix A must be expressible in a biorthogonal expansion having nonnegative coordinates with respect to extreme directions of some implicit pointed closed convex cone \mathcal{K} (§2.13.2.0.1, §2.13.7.1.1), or comparison to the origin with respect to some implicit pointed closed convex cone (2.7.2.2), or (when $\mathcal{K} = \mathbb{S}_+^n$) matrix A belongs to the positive semidefinite cone of symmetric matrices (§2.9.0.1), or (when $\mathcal{K} = \mathbb{R}_+^n$) vector A belongs to the nonnegative orthant (each vector entry is nonnegative, §2.3.1.1)
\succ	strict generalized inequality, membership to cone interior
$\not\succeq$	not positive definite
\geq	scalar inequality, greater than or equal to; comparison of scalars, or entrywise comparison of vectors or matrices with respect to \mathbb{R}_+
nonnegative	for $\alpha \in \mathbb{R}^n$, $\alpha \succeq 0$
$>$	greater than
positive	for $\alpha \in \mathbb{R}^n$, $\alpha \succ 0$; <i>id est</i> , no zero entries when with respect to nonnegative orthant, no vector on boundary with respect to pointed closed convex cone \mathcal{K}
$\lfloor \]$	floor function, $\lfloor x \rfloor$ is greatest integer not exceeding x
$ \ $	entrywise absolute value of scalars, vectors, and matrices
log	natural (or Napierian) logarithm
det	matrix determinant
$\ x\ $	vector 2-norm or Euclidean norm $\ x\ _2$

$$\begin{aligned}
\|x\|_\ell &= \sqrt[\ell]{\sum_{j=1}^n |x_j|^\ell} && \text{vector } \ell\text{-norm for } \ell \geq 1 && \text{(convex)} \\
&\triangleq \sum_{j=1}^n |x_j|^\ell && \text{vector } \ell\text{-norm for } 0 \leq \ell < 1 && \text{(violates §3.3 no.3)} \\
\|x\|_\infty &= \max\{|x_j| \mid \forall j\} && \text{infinity-norm} \\
\|x\|_2^2 &= x^T x = \langle x, x \rangle && \text{Euclidean norm square} \\
\|x\|_1 &= \mathbf{1}^T |x| && \text{1-norm, dual infinity-norm} \\
\|x\|_0 &= \mathbf{1}^T |x|^0 \quad (0^0 \triangleq 0) && \text{0-norm (§4.5.1), cardinality of vector } x \text{ (card } x) \\
\|x\|_k^n &&& \textit{k-largest norm (§3.3.2.1)} \\
\|X\|_2 &= \sup_{\|a\|=1} \|Xa\|_2 = \sigma_1 = \sqrt{\lambda(X^T X)_1} \text{ (615) matrix 2-norm (spectral norm),} \\
&&& \textit{largest singular value, } \|Xa\|_2 \leq \|X\|_2 \|a\|_2 \text{ [155, p.56], } \|\delta(x)\|_2 = \|x\|_\infty \\
\|X\|_2^* &= \mathbf{1}^T \sigma(X) && \textit{nuclear norm, dual spectral norm} \\
\|X\| &= \|X\|_F && \textit{Frobenius' matrix norm}
\end{aligned}$$